

The effect of rotation on the heat transfer between two nanoparticles

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Quantizing the electromagnetic vacuum and medium fields of two nanoparticles, we investigate the heat transfer between them. One of the particles has been considered to rotate with angular velocity ω_0 . The effect of rotation on the absorbed heat power by the rotating nanoparticle is discussed. While the results for angular velocities much smaller than the relaxation frequency Γ of the dielectrics are in agreement with the static nanoparticles, the increase of the angular velocity ω_0 in compare to the relaxation frequency of the dielectrics ($\omega_0 \geq \Gamma$) will generate two sidebands in the spectrum of the absorbed heat power. The well-known near-field and far-field effects are studied and it is shown that the sidebands peaks in the far-field are considerable in compare to the main peak frequency of the spectrum.

Development of nanotechnology in a wide variety of physical, chemical, biological, and medical context have raised a lot of open questions, especially in the case of rotating nanoparticles (NPs). Using rotating NPs for targeting cancer cells could be one of the most important applications of them and attracts a lot of interests [1–3]. Trapping and rotating NPs have been studied intensely using different methods [4–6]. Besides the important biomedical applications of rotating NPs, the effect of them also considered in some other cases e.g, on the instability of dust-acoustic waves [7].

Heat transfer in the nanoscale has been studied in a variety of nanofluids [8, 9], nano to macro scales [10], systems of a plane surface and NPs [11–13], between two NPs [14], between moving bodies [15], and two parallel metallic surfaces [16]. As the rotation of nanoparticles is getting important, one can ask, how does the rotation affect the heat transfer of NPs? It has been shown that the heat transfer absorbed by rotating NP from a plane surface could be changed [11].

The aim of this work is to find the effect of rotation on the heat transfer between two NPs. To this aim, we consider a system of two NPs where one located at the origin and the other one rotating along its axis of symmetry (axis z) and located on axis z a distance d from the origin (Fig.1).

Using the canonical field quantization approach, we find the explicit form of the quantized electromagnetic and dielectric fields in the non-relativistic regime, then the heat power absorbed by rotating NP is easy to drive in this scheme. A general formula for the heat power absorbed by a rotating NP from a static NP is obtained and the effect of rotation is discussed.

The Lagrangian describing the whole system contain a term represent the electromagnetic vacuum field plus terms modelling the dielectrics and their interaction with the electromagnetic vacuum field. following the method introduced in [11, 17], we study the heat transfer to the rotating NP and its physical consequences.

We consider the following Lagrangian for the mentioned system,

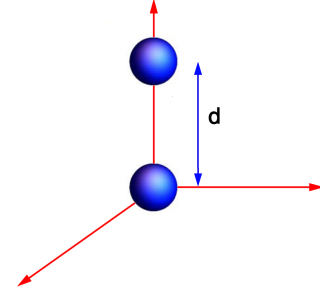


FIG. 1: A rotating nanoparticle located on axis z a distance d from a same static NP at the origin.

$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2}\epsilon_0 (\partial_t \mathbf{A})^2 - \frac{1}{2\mu_0} (\nabla \times \mathbf{A})^2 \\
 & + \frac{1}{2} \int_0^\infty d\nu [(\partial_t \mathbf{X}^1 + \omega_0 \partial_\varphi \mathbf{X}^1)^2 - \nu^2 (\mathbf{X}^1)^2] \\
 & - \epsilon_0 \int_0^\infty d\nu f_{ij}(\nu, t) X_j^1 \partial_t A_i \\
 & + \epsilon_0 \int_0^\infty d\nu f_{ij}(\nu, t) X_j^1 (\mathbf{v} \times \nabla \times \mathbf{A})_i \\
 & + \frac{1}{2} \int_0^\infty d\nu [(\partial_t \mathbf{X}^2)^2 - \nu^2 (\mathbf{X}^2)^2] \\
 & - \epsilon_0 \int_0^\infty d\nu f_{ij}(\nu, 0) X_j^2 \partial_t A_i.
 \end{aligned} \tag{1}$$

where X^1 and X^2 are the dielectric fields describing the first (rotating) and second NPs respectively and ω_0 is the angular velocity of the rotating NP. NPs are considered to be in local thermodynamical equilibrium at temperatures T_1 and T_2 respectively. $f_{ij}(\nu, t)$ is the coupling tensor between the electromagnetic vacuum field and the medium fields X^1 and X^2 . As the NPs are considered to be totally same, they should have same coupling tensor, while to take care of the rotation of NP.1, its coupling tensor $f_{ij}(\nu, t)$ is considered to be time dependent and

for NP.2 it is time-independent as $f_{ij}(\nu, t=0)$ where,

$$f_{ij}(\nu, t) = \begin{pmatrix} f_{xx}(\nu) \cos(\omega_0 t) & f_{xx}(\nu) \sin(\omega_0 t) & 0 \\ -f_{yy}(\nu) \sin(\omega_0 t) & f_{yy}(\nu) \cos(\omega_0 t) & 0 \\ 0 & 0 & f_{zz}(\nu) \end{pmatrix}.$$

To drive the Lagrangian Eq.(1), the coordinate derivative and field transformation between rotating and fixed frames are used. As the electromagnetic fields are non-rotating fields, there is no need to modify them.

The response function $\chi_{kk}^0(\omega)$, correspond to setting $\omega_0 = 0$, can be obtained in terms of the diagonal components of the coupling tensor $f_{ij}(\nu, t)$ as [17],

$$\chi_{kk}^0(\omega) = \epsilon_0 \int_0^\infty d\nu \frac{f_{kk}^2(\nu)}{\nu^2 - \omega^2}. \quad (2)$$

The response functions of the rotating NP in the laboratory frame can be written in terms of $\chi_{kk}^0(\omega)$,

$$\begin{aligned} \chi_{zz}^1(\omega, m) &= \chi_{zz}^0(\omega - m\omega_0), \\ \chi_{xx}^1(\omega, m) &= \chi_{yy}^1(\omega, m) \\ &= \frac{1}{2}[\chi_{xx}^0(\omega_+ - m\omega_0) + \chi_{xx}^0(\omega_- - m\omega_0)], \\ \chi_{xy}^1(\omega, m) &= -\chi_{yx}^1(\omega, m) \\ &= \frac{1}{2i}[\chi_{xx}^0(\omega_+ - m\omega_0) - \chi_{xx}^0(\omega_- - m\omega_0)], \end{aligned} \quad (3)$$

where $\omega_\pm = \omega \pm \omega_0$.

Defining $P_i^k(\mathbf{r}, t) = \epsilon_0 \int_0^\infty d\nu f_{ij}(\nu, t) X_j^k(\mathbf{r}, t, \nu)$, as the electric polarization components of NPs, we obtain the equations of motion for the electromagnetic and matter fields as,

$$\begin{aligned} \mathbf{P}^1(\mathbf{r}, \omega) &= \mathbf{P}^{N,1}(\mathbf{r}, \omega) + \epsilon_0 \chi^1(\omega, -i\partial_\varphi) \mathbf{E}, \\ \mathbf{P}^2(\mathbf{r}, \omega) &= \mathbf{P}^{N,2}(\mathbf{r}, \omega) + \epsilon_0 \chi^0(\omega) \mathbf{E}, \\ \left\{ \nabla \times \nabla \times - \frac{\omega^2}{c^2} \mathbb{I} - \frac{\omega^2}{c^2} \chi^1(\omega, -i\partial_\varphi) - \frac{\omega^2}{c^2} \chi^0(\omega) \right\} \cdot \mathbf{E} \\ &= \mu_0 \omega^2 (\mathbf{P}^{N,1} + \mathbf{P}^{N,2}), \end{aligned} \quad (4)$$

where $\mathbf{P}^{N,1}$ and $\mathbf{P}^{N,2}$ are the fluctuating or noise electric polarizations correspond to the fluctuating or noise matter fields $\mathbf{X}^{N,1}$ and $\mathbf{X}^{N,2}$ for the rotating and static NPs respectively. One can expand them in terms of ladder operators as,

$$\begin{aligned} X_i^{N,1}(\rho, \varphi, z, \nu, t) &= \sum_m [e^{im\varphi} e^{i(\nu - m\omega_0)t} a_{i,m}^\dagger(\rho, z, \nu) \\ &\quad + e^{-im\varphi} e^{-i(\nu - m\omega_0)t} a_{i,m}(\rho, z, \nu)], \\ X_i^{N,2}(\mathbf{r}, \nu, t) &= e^{i\nu t} b_i^\dagger(\mathbf{r}, \nu) + e^{-i\nu t} b_i(\mathbf{r}, \nu). \end{aligned} \quad (5)$$

In case of holding the NPs in thermal equilibrium at temperature T_1 and T_2 , we have

$$\begin{aligned} \langle a_{i,m}^\dagger(\rho, z, \nu) a_{j,m'}(\rho', z', \nu') \rangle_T &= \frac{\hbar}{4\pi\nu} n_{T_1}(\nu) \delta_{mm'} \delta_{ij} \\ \delta(\nu - \nu') \frac{\delta(\rho - \rho') \delta(z - z')}{\rho}, \\ \langle b_i^\dagger(\mathbf{r}, \nu), b_j(\mathbf{r}', \nu') \rangle_T &= \frac{\hbar}{2\nu} n_{T_2}(\nu) \delta_{ij} \delta(\nu - \nu') \delta(\mathbf{r} - \mathbf{r}'), \end{aligned} \quad (6)$$

where $n_T(\omega) = [\exp(\hbar\omega/kT) - 1]^{-1}$.

Using (4) and the dyadic Green's tensor G_{ij} , we find

$$\begin{aligned} E_i(\mathbf{r}, \omega) &= E_i^0(\mathbf{r}, \omega) + \mu_0 \omega^2 \int_{V_1} d\mathbf{r}' G_{ij}(\mathbf{r}, \mathbf{r}', \omega) P_j^{N,1}(\mathbf{r}', \omega) \\ &\quad + \mu_0 \omega^2 \int_{V_2} d\mathbf{r}' G_{ij}(\mathbf{r}, \mathbf{r}', \omega) P_j^{N,2}(\mathbf{r}', \omega), \end{aligned} \quad (7)$$

where the first term on the right-hand side of (7) corresponds to the fluctuations of the electric field in electromagnetic vacuum, while the second and third terms are the induced electric field due to the fluctuations of the electric polarization of the NPs.

The rate of work done by the electromagnetic field on a differential volume $d\mathbf{r}$ of a dielectric is given by $\mathbf{j} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) d\mathbf{r}$ where $\mathbf{j} = \partial_t \mathbf{P} - \nabla \times (\mathbf{v} \times \mathbf{P})$ is the current density in matter. In non-relativistic regime, we ignore the terms containing velocity \mathbf{v} ; therefore, the radiated power of the rotating NP can be written as

$$\langle \mathcal{P} \rangle = \int_V d\mathbf{r} \int \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} e^{-i(\omega + \omega')t} (i\omega) \langle \mathbf{P}(\mathbf{r}, \omega) \cdot \mathbf{E}(\mathbf{r}, \omega') \rangle. \quad (8)$$

The radiated power $\langle \mathcal{P} \rangle$ contains all emitted and absorbed energy of the rotating NP due to the interaction with electric field \mathbf{E} . One of the terms contains the fluctuating electrical polarization of the static NP at the origin $\langle \mathbf{p}_i^{N,2}(\mathbf{r}, \omega) \cdot \mathbf{p}_j^{N,2}(\mathbf{r}', \omega') \rangle$, which we will focus on in the following, is responsible for the heat transfer power $\langle \mathcal{P} \rangle_{HT}$ from the static NP at the origin to the rotating NP on the axis z a distance d from the origin. Using Eqs. (7), and (8), we drive

$$\begin{aligned} \langle \mathcal{P} \rangle_{HT} &= \frac{2\hbar}{\pi} \int_0^\infty d\omega \frac{\omega^5}{c^4} \text{Im}[\alpha_{ij}^1(\omega)] \text{Im}[\alpha_{kk}^0(\omega)] n_{T_2}(\omega) \\ &\quad \times \int d\mathbf{r}' G_{ik}(\mathbf{r}, \mathbf{r}', \omega) G_{kj}^*(\mathbf{r}, \mathbf{r}', \omega). \end{aligned} \quad (9)$$

where $\text{Im}[\alpha_{ij}(\omega)] = V \text{Im}[\chi_{ij}(\omega, m=0)]$ and V represents the volume of NPs. One can find a proper dyadic Green's tensor G_{ij} for Eq.(7) as,

$$\begin{aligned} G_{ij}(\mathbf{r}, \mathbf{r}', \omega) &= \frac{e^{ikR}}{R^3 k^2} [(k^2 R^2 + ikR - 1) \delta_{ij} \\ &\quad - (k^2 R^2 + 3ikR - 3) \frac{R_i R_j}{R^2}], \end{aligned} \quad (10)$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $k = \omega/c$. In this work, the inter-particle distance d between NPs are chosen to be much bigger than their radius, therefore, it is a good approximation to consider them as point-like particles. On the other hand, using this approximation, the components of dyadic Green's tensor G_{ij} can be simplified to,

$$\begin{aligned} G_{xx}(0, d\hat{z}, \omega) &= G_{yy}(0, d\hat{z}, \omega) = \frac{e^{ikd}}{d^3 k^2} (k^2 d^2 + ikd - 1), \\ G_{zz}(0, d\hat{z}, \omega) &= \frac{2e^{ikd}}{d^3 k^2} (1 - ikd), \end{aligned} \quad (11)$$

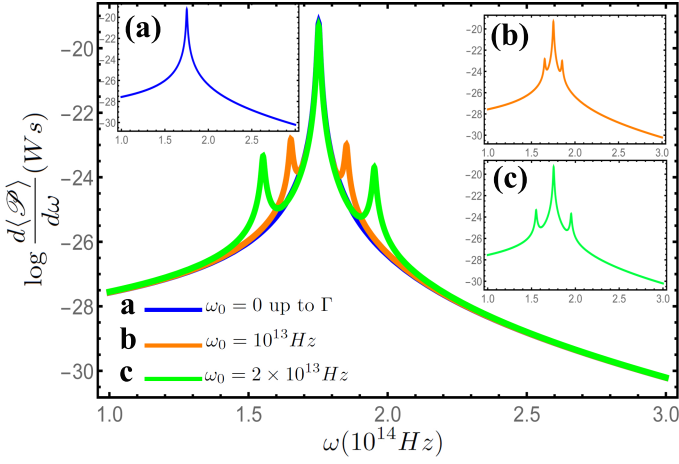


FIG. 2: (Color online) Logarithmic (base 10) plot of the absorbed heat power spectrum of a rotating NP located on the axis z a distance $d = 10nm$ from a static NP at the origin as a function of frequency where the static NP is considered to be at room temperature ($T = 300k$).

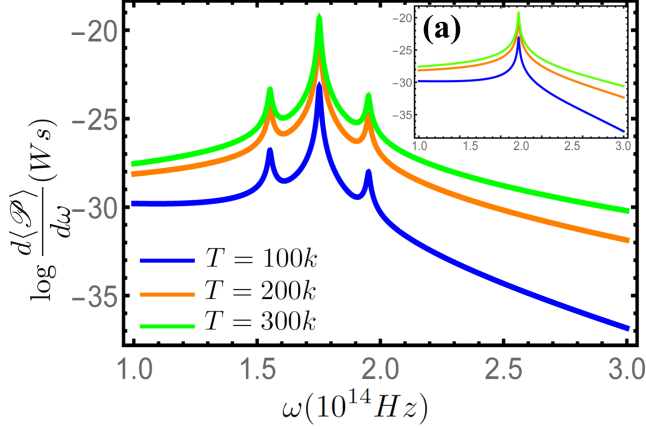


FIG. 3: (Color online) Logarithmic (base 10) plot of the absorbed heat power spectrum of a rotating NP $\omega_0 = 2 \times 10^{13}$, (a) a non-rotating NP, located on the axis z a distance $d = 10nm$ from a static NP at the origin as a function of frequency for different temperature of static NP.

and all other components are vanished.

To find some numerical results, the NPs are considered to be made of Silicon Carbide (SiC) with same radius ($a = 2nm$), where the dielectric function is given by the oscillator model [18],

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 + \frac{\omega_L^2 - \omega_T^2}{\omega_T^2 - \omega^2 - i\Gamma\omega} \right), \quad (12)$$

with $\varepsilon_\infty = 6.7$, $\omega_L = 1.823 \times 10^{14}$, $\omega_T = 1.492 \times 10^{14}$, and $\Gamma = 8.954 \times 10^{11}$.

It has been seen that in a cavity optomechanics, a mirror oscillating with frequency ω_0 , the optical sidebands are created around the incoming light frequency ω ,

$$\omega' = \omega \pm \omega_0 \quad (13)$$

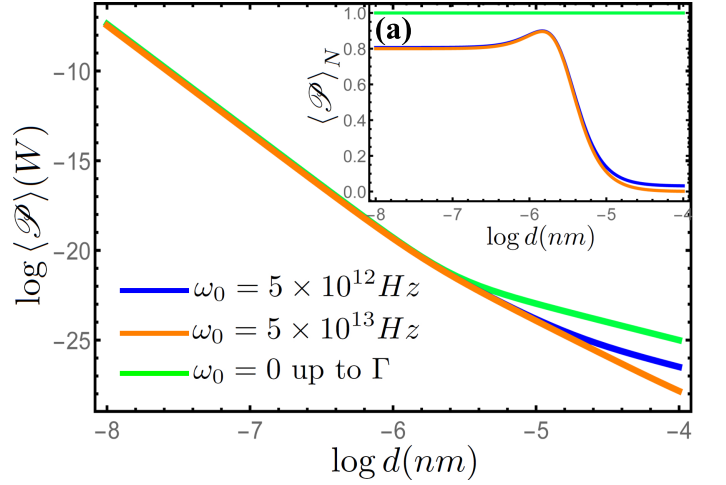


FIG. 4: (Color online) Logarithmic (base 10) plot of total absorbed heat power by the rotating NP with angular velocity ω_0 , (a) total absorbed heat power by the rotating NP with angular velocity ω_0 divided by the total absorbed heat power of non-rotating NP, as a function of the interparticle distance d where the static NP is considered to be at room temperature ($T = 300k$).

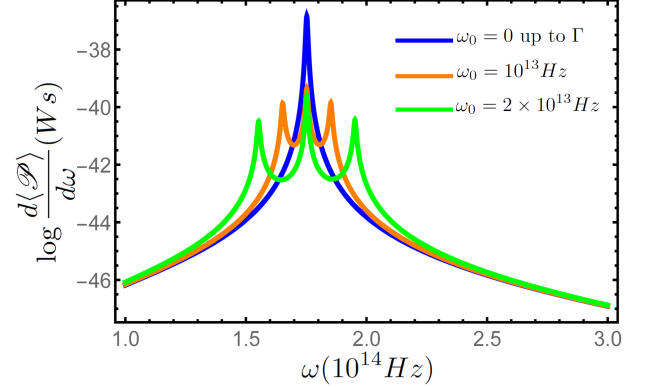


FIG. 5: (Color online) Logarithmic (base 10) plot of the absorbed heat power spectrum of a rotating NP located on the axis z a distance $d = 100\mu m$ from a static NP at the origin as a function of frequency where the static NP is considered to be at room temperature ($T = 300k$).

although, in this work we are far from the cavity optomechanics, but both cases contain a kind of oscillation frequency ω_0 on the matter fields. Thus, one can expect same results on the spectrum of the absorbed heat transfer, where interestingly, it is supported by Fig. (2). It shows that, increasing the angular velocity ω_0 , can cause a couple of sidebands on the spectrum of the absorbed heat transfer by rotating NP from the static NP, where the sidebands appeared around the remarkable peak frequency ω of the absorbed heat transfer spectrum of non-rotating NPs. The sidebands frequencies are given by Eq. (13). While the spectrum, for angular velocities smaller than the relaxation frequency Γ of dielectrics, is as same

as the spectrum for non-rotating NPs.

The effect of static NP temperature T_2 on the spectrum of the absorbed heat transfer of rotating NP has been depicted in Fig. (3). As a result of that, the temperature of the static NP will affect the absorbed heat transfer of rotating and non-rotating NP in the same way.

To focus more on the effect of rotation in the near and far field of the static NP, the total absorbed heat power of rotating NP as a function of the interparticle distance d has been depicted in Fig. (4), where it provides a dependence on d^{-6} in the near field as reported previously for non-rotating NPs [14]. Fig. (4b) shows a transition on the total absorbed heat power of rotating NP normalized by the total absorbed heat power of a non-rotating NP. This is a consequence of transition between near field and far field of the static NP. One can conclude, the effect of rotation on the absorbed heat power of a rotating NP can be quite different in near field and far field of a static NP. Therefore, we renew the plot of Fig. (2), where it was depicted in the near field, in far field of the static NP. In Fig. (5) surprisingly, the contribution of sidebands peaks has raised in compare to the main peak of the spectrum. This future may find a lot of applications e.g in designing the new optomechanical systems.

In conclusion, the effect of rotation on the heat transfer between two NPs has been analysed. Two sidebands appeared around the peak frequency of the heat transfer spectrum of non-rotating NPs. While the peaks of the sidebands are so small in compare to the main peak in the near field, they are considerable in the far field even up to the same order of magnitude in compare to the main peak frequency of the spectrum.

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